Friedrich-Alexander-Universität
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Null Controllability for Population Dynamics with age, size Structuring and Diffusion

MINI WORKSHOP FAU DCN-AvH

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1. Outline

2. Motivation and description of age and size structured model

2.1 Motivation

2.2 Description of age and size structured model

3. Observation and null controllability

3.1 Null controllability

4. Perspectives

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Motivation

1. In this talk, we are interested of the null controllability of population dynamics with age, size Structuring and Diffusion. A population dynamics model that can describe cell growth and more precisely carcinogenic growth.
   Many authors have been interested in studying the well posedness of these models; but very few are interested in the control aspect.

Motivation

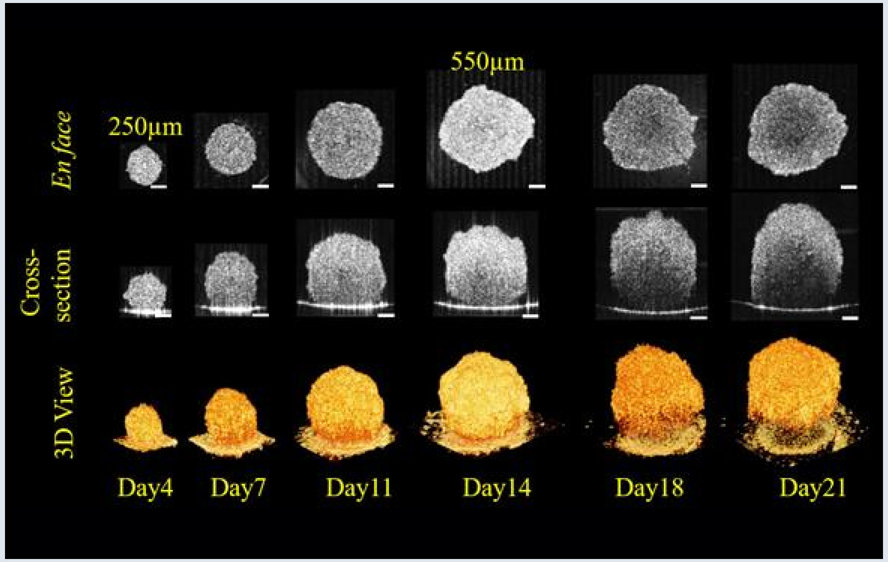
1. In this talk, we are interested of the null controllability of population dynamics with age, size Structuring and Diffusion.
   A population dynamics model that can describe cell growth and more precisely carcinogenic growth.
   Many authors have been interested in studying the well posedness of these models; but very few are interested in the control aspect.
2. 

Figure: Tumor Growth: Imaging technology could better monitor tumor growth, drug effectiveness

Description of age and size structured model

$$
\begin{cases}
\dfrac{\partial y}{\partial t} + \dfrac{\partial y}{\partial a} + \dfrac{\partial (g(s)y)}{\partial s} - \Delta y + \mu(a,s)y = 0 & \text{in } \Omega \times (0,A) \times (0,S) \times (0,+\infty), \\
\dfrac{\partial y}{\partial \nu} = 0 & (x,a,s,t) \in \partial\Omega \times (0,A) \times (0,S) \times (0,\infty), \\
y\,(x,0,s,t) = y_i(x,s,t) & (x,s,t) \in \Omega \times (0,S) \times (0,\infty) \\
y\,(x,a,s,0) = y_0\,(x,a,s) & (x,a,s) \in \Omega \times (0,A) \times (0,S); \\
y(x,a,0,t) = 0 & (x,a,t) \in \Omega \times (0,A) \times (0,\infty).
\end{cases}
\tag{1}
$$

Here $y(x,a,s,t)$ (carcinogenic cells) is a distribution of individuals of age a size s at time t and location $x \in \Omega$. A and S are respectively the maximal live expectancy and the maximal size, $\mu(a,s)$ natural death rate of individuals of individual.

1. $\dfrac{\partial y}{\partial a}$ is the aging
2. $\dfrac{\partial y}{\partial s}$ is the growing
3. $\mu(a,s)y$ is the damping
4. $y_i(x,s,t)$, $i \in \{1,2\}$ is the birth the rate, here we consider two type of birth rate.
   - 4.1 $y_2(x,s,t) = \int_0^A \beta_2(a,s)y(x,a,s,t)da$, where β_2 is the fertility function;
   - 4.2 $y_1(x,s,t) = \int_0^A \int_0^S \beta_1(a,\hat{s},s)y(x,a,\hat{s},t)dad\hat{s}$, where the fertility β_1 can mean the probability of an individual of age a and of size $\hat{s}$ giving birth to an individual of size s.

Description of age and size structured model

According to Gleen Webb, if the mortality and fertility rates $\mu(a,s)=\mu_1(a)+\mu_2(s)$ and β_i are such that:

$$(\mathbf{H1}):\begin{cases}\mu_1(a)\geq 0 \text{ for every } a\in(0,A)\\ \mu_1\in L^1([0,a^*]) \text{ for every } a^*\in[0,A)\\ \int_0^A \mu_1(a)da=+\infty\end{cases},\quad (\mathbf{H2}):\begin{cases}\mu_2(s)\geq 0 \text{ for every } s\in(0,S)\\ \mu_2\in L^1([0,s^*]) \text{ for every } s^*\in[0,A)\\ \int_0^S \mu_2(s)ds=+\infty\end{cases}$$

$$(\mathbf{H3}):\begin{cases}\beta_i\in L^\infty,\ i\in\{1,2\}\\ \beta_i\geq 0 \quad \text{a.e.} \quad i\in\{1,2\},\end{cases}$$

for any initial condition $y_0\in K=L^2(\Omega\times(0,A)\times(0,S))$, the system (1) admits a unique solution.

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The Null controllability problem

$$\begin{cases} \dfrac{\partial y}{\partial t}+\dfrac{\partial y}{\partial a}+\dfrac{\partial (g(s)y)}{\partial s}-\Delta y+\mu(a,s)y=u\chi_{\Theta} & \text{in } \Omega\times(0,A)\times(0,S)\times(0,\infty), \\ \dfrac{\partial y}{\partial \nu}=0 & (x,a,s,t)\in\partial\Omega\times(0,A)\times(0,S)\times(0,\infty), \\ y\left(x,0,s,t\right)=y_i(x,s,t) & (x,s,t)\in\Omega\times(0,S)\times(0,\infty),\ i\in\{1,2\} \\ y\left(x,a,s,0\right)=y_0\left(x,a,s\right) & (x,a,s)\in\Omega\times(0,A)\times(0,S); \\ y(x,a,0,t)=0 & (x,a,t)\in\Omega\times(0,A)\times(0,\infty). \end{cases} \tag{2}$$

where $\Theta=\omega\times(a_1,a_2)\times(s_1,s_2)$

1. $u(x,a,s,t)$ is the control;
2. $\omega\times(a_1,a_2)\times(s_1,s_2)\subset\Omega\times(0,A)\times(0,S)$ is the support of the control;
3. Goal: To drive the solution to equilibrium at a given final time $T>0$

$$y(.,.,.,T)\equiv 0$$

The dual observation

Consider the adjoint system:

$$\begin{cases} \frac{\partial q}{\partial t} - \frac{\partial q}{\partial a} - \frac{\partial q}{\partial s} - \Delta q + \mu(a,s)q = q_i(x,s,t) & \text{in } \Omega \times (0,A) \times (0,S) \times (0,+\infty) i \in \{1,2\}, \\ \frac{\partial q}{\partial \nu} = 0 & (x,a,s,t) \in \partial\Omega \times (0,A) \times (0,S) \times (0,\infty), \\ q(x,A,s,t) = 0 & \text{in } (x,s,t) \in \Omega \times (0,S) \times (0,\infty) \\ q(x,a,S,t) = 0 & \text{in } (x,a,t) \in \Omega \times (0,A) \times (0,\infty) \\ q(x,a,s,0) = q_0(x,a,s) & \text{in } (x,a,s) \in \partial\Omega \times (0,A) \times (0,S); \end{cases} \tag{3}$$

with the following correspondence

$$q_1(x,s,t) = \int_0^S \beta_1(a,s,\hat{s})q(x,0,\hat{s},t)d\hat{s} \text{ matches with } y_1, \text{ and } q_2(x,s,t) = \beta_2(a,s)q(x,0,s,t) \text{ matches with } y_2.$$

The question is whether:

$$\int_0^S \int_0^A \int_\Omega q^2(x,a,s,T)dxdads \le K_T \int_0^T \int_\Theta q^2(x,a,s,t)dxd\Theta. \tag{4}$$

Null controllability results

We denote by

$$T_1 = \max\{a_1 + S - s_2, s_1\} \text{ and } T_0 = \max\{S - s_2, s_1\}.$$

Theorem

The null controllability result holds in

$$\omega \times (a_1, a_2) \times (s_1, s_2) \subset \Omega \times (0, A) \times (0, S)$$

with $T_0 < \min\{a_2 - a_1, \gamma\}$; *provided the fertility rate is such that*

$$\beta_i(a, .) \equiv 0 \text{ in } (0, a_1 + \gamma),$$

and the time T is large enough as follows:

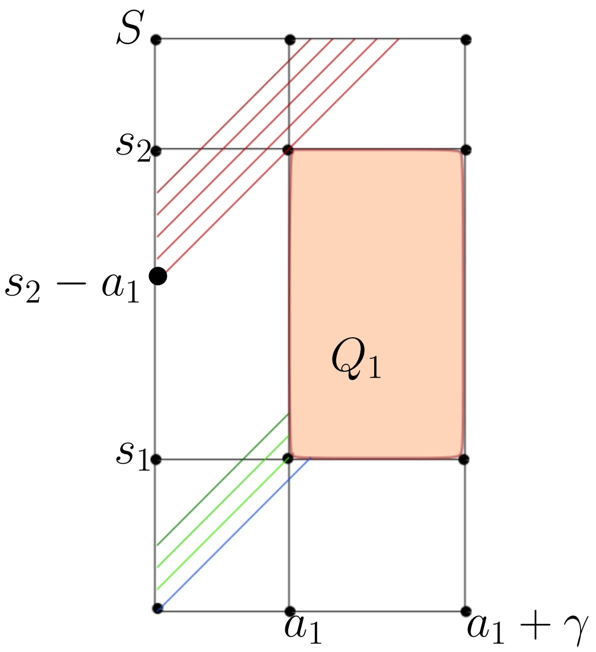
1. *for the birth rate equal* y_1, $T > A - a_2 + T_1 + T_0$ *and*
2. *for the birth rate equal* y_2, $T > A - a_2 + a_1 + S - s_2 + s_1$.

See [Bic22][SIM22].

Graphical proof of the estimation of $q(x, 0, s, t)$

As $\beta_i(a, .) \equiv 0$ in $(0, a_1 + \gamma)$, the first equation of the adjoint system become.

$$\frac{\partial q}{\partial t} - \frac{\partial q}{\partial a} - \frac{\partial q}{\partial s} - \Delta q + \mu(a, s)q = 0 \text{ in } \Omega \times (0, a_1 + \gamma) \times (0, S) \times (0, T). \tag{5}$$



$T_0 < \min\{a_2 - a_1, \gamma\}$ we choose $a_2 = a_1 + \gamma$. And we need to estimate

$$q(x, 0, s, t)$$

If $s \in (0, s_2, -a_1)$ and $t > \max\{a_1, s_1\}$ all the backward characteristics starting from $(0, s, t)$ enters the observation domain and if $s \in (s_2 - a_1, S)$ and $t > a_1 + S - s_2$(the green and blue lines), all the backward characteristics starting from $(0, s, t)$ without the domain by the boundary $s = S$ (red line) and then $q(x, 0, s, t) = 0$.

Proof of observability inequality: Estimation of q at finale time T

1. For the birth rate y_2, we can split the section $t = T$ by two section $U_1 = [0, A] \times [0, s_1 - a_1] \times \{T\}$ and $U_2 = [0, A] \times [s_2 - a_1, S] \times \{T\}$ and we use the previous estimation of $q(x, 0, s, t)$ on $(0, s_2 - a_1)$ and $(s_2 - a_1, S)$ to obtain the observability inequality in time $T > A - a_2 + S - s_2 + a_1 + s_1$.
2. For the birth rate y_1, we can not split the section $t = T$ because the second member of the adjoint system is the sum of $q(x, 0, s, t)$ with respect to the size variable given by q_1 then, we have the estimation of q_1 if $t > T_1$ and then observability inequality is true if $T > A - a_2 + T_0 + T_1$.

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Perspectives

- Extend the study to:

$$\frac{\partial y}{\partial t} + \frac{\partial y}{\partial a} + \frac{\partial (g(s)y)}{\partial s} - Ly + \mu(a,s)y = 0$$

  where

$$Ly = \alpha(.)\Delta y - \nabla.\left(\chi(.)y\right)$$

- Modeling, control and simulation of tumour growth

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[Bic22] S. Y. U. Biccari. “Controllability and Positivity Constraints in Population Dynamics with age, size Structuring and Diffusion”. In: (2022).

[SIM22] U. B. SIMPORE Yacouba Yassine El gantouh. “Null Controllability for a Degenerate Structured Population Model”. In: (2022).


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謝謝
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Баярлалаа
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kia ora
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blagodaram
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misaotra
matondo
paldies
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thank you
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хвала
asante
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